Neutrino Oscillations and Solar Neutrino Problem

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Abstract

I review neutrino oscillations in vacuum and matter. In vacuum oscillations, wave packets as well as plane wave states are discussed. I show that neutrinos oscillation disappears after neutrinos travel far long becase the coherence between two wave packets are lost. In matter oscillations, I discuss variable density case after simple constant density case. I also show the adiabatic conditions derived by Landau and Zener, which is accurate unless mixing angle close to $\frac{\pi}{4}$. At the end, I introduce the solar neutrino problem, which was firstly observed by the Cl experiment, followed by Davis, Jr. By many experiments like Super-Kamiokande, SNO and KamLAND, it has been understood that the solar neutrino problem could be explained by the MSW effect.

Contents

Chapter 1

Introduction

In the Standard Model, which is a mathematical description of the strong, weak, and electromagnetic interaction, neutrinos were introduced as truly massless fermions for which no gauge invariant renormalizable mass term can be constructed. But recently Solar neutrino and Atmospheric neutrino experiments show an evidence which neutrino have mass. If neutrino have mass, it occers flavor change and 'neutrino oscillation' with travering vacuum or matter. It is a evidence that neutrinos have masses.

In second capture I introduce a neutrino propaties in the standard model. In third capture I discuss neutrino oscillation in vacuum with two generation approximation. In forth capture I discuss neutrion oscillation in matter. When neutrinos propagate in dense matter, the interactions with the medium affect their propaties. So neutrino energy receivs an extra contribution (effective potencial). Consecently, wederive MSW effect and appropriate the effect Solar neutrion problem. In fifth capture I introduce sevelal experiments of solar neutrion in the world, Homestake, Kamiokande, Sno. There experiment results show the evidence of neutrino oscillation and we derive an information of oscillation parameters, mass diference and mixing angle. Having neutrino mass provides an unambiguous sugnal for new physics.

Chapter 2

Properties of Neutrinos

2.1 History of Neutrino

In 1930 Wolfgang Pauli postulated the existence of the neutrino in order to reconcile data on the radioactive decay of nuclei with energy conservation. In radioactive decays, nuclei of atoms mutate into different nuclei when neutrons are transformed into slightly lighter protons with the emission of electrons:

$$\text{neutron} \to \text{proton} + \text{electron} + \text{antineutrino}. \tag{2.1}$$

Without the neutrino, energy conservation requires that the electron and proton share the neutron's energy. Each electron is therefore produced with a fixed energy while experiments indicated conclusively that the electrons were not mono-energetic but observed with a range of energies. This energy range coressponded exactly to the many ways the three particles in the final states of the reaction above can share energy while satisfying its conservation law. The postulated neutrino had no electric charge. It just serves as an agent to balance energy and momentum in above reaction. In fact, Pauri pointed out that for the neutrino to do the job, it had to weigh less than one percent of the proton mass.

Observing neutrinos is straightfoward. Quater of a century later, Fled Reines and Clyde Cowan Jr, observed neutrinos produced by a nuclear reactor. In the presence of protons, neutrinos occasionary initiate the inverse reaction of radioactive decay,

$$\nu + p \to n + e^+ \tag{2.2}$$

Neutrinos are also produced in natural sources. Starting at the 1960's, neutrinos produced in the sun and in the atmosphere have been observed. In 1987, neutrinos from a supernova in the Large Magellanic Cloud were also detected.

Newtrino is a lepton that have no charge and few mass. So it interacts with other particle in weak interaction and gravity interaction.

Neutrino has three generations. All species of neutrinos are equally produced in some high-energy reactions. The most definitive constraint is derived from the total width of Z^0. Z^0 decays into e^+e^-, $\mu^+\mu^-$, $\tau^+\tau^-$, $q\bar{q}(q = u, d, s, c, b)$ and $\nu\bar{\nu}'s$. Since e, μ, τ and q has a charge, the decays visible. But $\nu's$ decay invisible. The partial width of Z^0 for invisible decays measured with LEP experiments yields

$$N_\nu = 2.994 \pm 0.012 \quad . \tag{2.3}$$

The result gives the number of neutrino species.

2.2 Neutrino in the Standard Model

In the labotarory neutrino masses have been seached for in two types of experiments: (1) direct kinematic seaches of neutrino mass, of which the most sensitive is the study of tritium beta decay, and (2) neutrinoless double-β decay experiments. Experiments achived higher and higher precision, reaching upper limits for the electron-neutrino mass of 10^{-9} the proton mass. This raised the question of whether neutrinos are truly massless like photons.

In 1957 Bruno Pontecorvo realized that the existance of neutrino masses implies the possibility of neutrino oscillations. This phenomenon is similar to what happens in the quark sector, where nertrino kaons oscellate. Flavor oscillations of neutrino have been searched for using either neutrino beams from reactor or accelerators, or natural neutrinos generated at astrophysical sources (the Sun giving the largest flux) or in the atmosphere (as the byproducts of cosmic ray collisions). The longer the distance that the neutrinos travel from their production point to the detector, the smaller masses that can be signaled by their oscillation. Indeed, the solar neutrinos allow us to search for masses that are as small as 10^{-5} eV, that is 10^{-14} of the proton mass.

quark	mass (MeV)	charge	lepton	mass (MeV)	charge
u	1.5∼4.0	2/3	e	0.51	-1
d	4∼8	-1/3	ν_e	0	0
c	$1.15\times10^3 \sim 1.35\times10^3$	2/3	μ	105.6	-1
s	80∼130	-1/3	ν_μ	0	0
t	$174.3\pm5.1 \times 10^3$	2/3	τ	1.77×10^3	-1
b	$4.6\times10^3 \sim 4.9\times10^3$	-1/3	ν_τ	0	0

Table 2.1: Charge and mass of leptons and quarks in Standard Model

What can we learn from measurment of neutrino masses about our theories of particle physics? The standard model (SM) of particle phsics is a mathematical description of the strong, weak and electromagnetic interactions. Since it was conceived in the 1960's by Glashow, Salam and Weinberg, it has successfully passed numerous experimental tests. In the absence of any direct evidence for their mass, neutrinos were introduced in the SM as truly massless fermions for which no gauge invariant renormalizable mass term can be constructed.

Chapter 3

Neutrino Oscillation in Vacuum

3.1 Oscillation Probability of plane wave neutrinos

Neutrino oscillations in vaccum would arise if neutrinos were massive and mixed. In other words, the neutrino state that is produced by electroweak interactions is not a mass eigenstates. This phenomenon was first pointed out by Pontecorvo in 1957 while the possibility of arbitrary mixing between two massive neutrino states was first introduced in Maki, Nakagawa and Sakata in 1962.

If neutrino have masses, the weak eigenstates, ν_α, produced in a weak interaction are sutisfied the Shrödinger equation.

$$i\frac{\partial}{\partial t}\,|\nu_f\rangle = H\,|\nu_f\rangle, \tag{3.1}$$

$$|\nu_f\rangle = \begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix} \tag{3.2}$$

H is hamiltonian and it satisfis $H = \sqrt{p^2 + m^2}$.(p^2 is a number and m^2 is a non diagnal matrix.) we write $p = E$ and approximate H

$$H \simeq E + \frac{m^2}{2E} \tag{3.3}$$

We diagnalize H by using an unitary matrix U as

$$U^\dagger H U = E + \frac{U^\dagger m^2 U}{2E}. \tag{3.4}$$

Since

$$U^\dagger m^2 U = {m^2}_{diag} = \begin{pmatrix} m_1^2 & & \\ & m_2^2 & \\ & & m_1^3 \end{pmatrix} \tag{3.5}$$

we may write (3.1) by using $U_{\alpha\beta}(U_{\alpha\beta})^\dagger = U_{\alpha\beta}(U^\dagger)_{\beta\alpha} = 1$

$$\begin{aligned} U^\dagger(i\frac{\partial}{\partial t}\left|\nu_f\right\rangle) &= U^\dagger(E + \frac{m^2}{2E})UU^\dagger\left|\nu_f\right\rangle \\ &= (E + \frac{m^2{}_{diag}}{2E})U^\dagger\left|\nu_f\right\rangle \end{aligned} \tag{3.6}$$

We define as

$$\left|\nu_i\right\rangle \equiv U^\dagger\left|\nu_f\right\rangle. \tag{3.7}$$

Hence the equation $\left|\nu_i\right\rangle$ satisfy is

$$i\frac{\partial}{\partial t}\left|\nu_i\right\rangle = (E + \frac{m^2_{diag}}{2E})\left|\nu_i\right\rangle, \tag{3.8}$$

using the approximation that $\left|\nu\right\rangle$ is a plane wave gives time dependence of $\left|\nu_i\right\rangle$ as

$$\left|\nu_i(t)\right\rangle = e^{-i(E+m_i)t}\left|\nu_i(0)\right\rangle. \tag{3.9}$$

Therefore a neutrino of the generation $\alpha(=e,\mu,\tau)$, after a time intervas of t, is given by

$$\begin{aligned} \left|\nu_\alpha(t)\right\rangle &= (U^\dagger)_{\alpha i}\left|\nu_i(t)\right\rangle \\ &= (U^\dagger)_{\alpha i}e^{-i(E+m_i)t}\left|\nu_i(0)\right\rangle. \end{aligned} \tag{3.10}$$

The transition amplitude that to the state to the state $\left|\nu_\beta\right\rangle$ is

$$\left\langle\nu_\beta|\nu_\alpha(t)\right\rangle = \sum_i U_{\alpha i}(U^\dagger)_{i\beta}e^{-iE_it}. \tag{3.11}$$

If we restrict ourselves to mixing between the two generations, we can parametrize U as

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}. \tag{3.12}$$

For example the trantision from $\left|\nu_e\right\rangle$ to $\left|\nu_e\right\rangle$ is

$$\left\langle\nu_e|\nu_e\right\rangle_t = \cos\left(\frac{\Delta m^2}{4E}t\right) - i\sin\left(\frac{\Delta m^2}{4E}t\right)\cos 2\theta \tag{3.13}$$

and its transition probability is

$$\begin{aligned} P_{\nu_e\to\nu_e} &= |\left\langle\nu_e|\nu_e\right\rangle_t|^2 \\ &= 1 - \sin^2\theta\sin^2\left(\frac{\Delta m^2}{4E}t\right), \end{aligned} \tag{3.14}$$

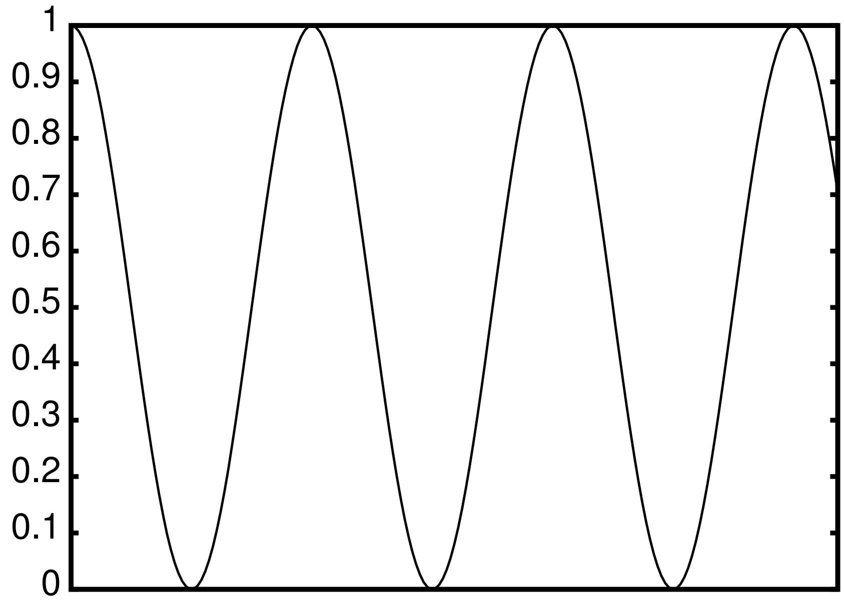

Figure 3.1: Transition Probability for $\nu_e \rightarrow \nu_e$ oscillation. The mixing angle θ is taken to be $\frac{\pi}{4}$.

and to $|\nu_\mu\rangle$ is

$$\begin{aligned} P_{\nu_e \rightarrow \nu_\mu} &= |\langle \nu_\mu | \nu_e \rangle|^2 \\ &= \sin^2\theta \sin^2\left(\frac{\Delta m^2}{4E} t\right). \end{aligned} \tag{3.15}$$

We call this probability oscillation 'neutrino oscillation'. In Fig.3.1, we show the $\nu_e \rightarrow \nu_e$ transition probability for $\theta = \pi/4$.

Asumming that neutrino propagates with light velocity ,we call l_0 which satisfies

$$\frac{\Delta m^2}{4E}(l_0/c) = \pi \tag{3.16}$$

the oscillation length. If we observe the neutrino flux at a distance $L(= ct)$ from the source, we find from (3.14),

$$P_{\nu_e \rightarrow \nu_e} = 1 - \sin^2 2\theta \sin^2\left(\pi \frac{L}{i_0}\right). \tag{3.17}$$

For $L \gg l_0$ or

$$\Delta m^2 \gg 4\pi E/L = \Delta m_m^2 in, \tag{3.18}$$

the second sine factor oscillates rapidly, so that

$$P_{\nu_e \rightarrow \nu_e} = 1 - \frac{1}{2}\sin^2 2\theta, \tag{3.19}$$

which stands for a reduction of the ν_e flux averaged over a sufficiently long time. This is often called 'time-averaged oscillation.'

3.2 Oscillation Probability of wavepacket neutrinos

In previous section we used the approximation that $|\nu\rangle$ is a plane wave. But in the following situation what effect we seen ? If the neutrino beam is localized at the origin, the neutrino clouds corresponding to the two different mass eigenstates move with different velocities and will be separated by an amount of $(\Delta v)ct \approx (\frac{\Delta m^2}{E^2}ct)$ after time t. If this separation is larger than the neutrino coherence length, the two clouds no longer overlap, and hence do not interfere. This means that neutrino oscillation eventually disappears.

This situation is clearly understood in the formuration using the wave packet. In order to construct the wave packets for the mass eigenstates, we make the following assumption: The ploblem is one dimensional. The mass-eigenstate wave packets have a Gaussian form with the same width σ_p in mamentum space. The Gaussian mass-eigenstate wave packets in momentum space are sharply peaked around the mean value $\bar{p}_i$.

The normalized mass-eigenstate wave packets in momentum and coordenate spaces, respectively, are given by

$$\psi_i(p) = \frac{1}{\sqrt{2\pi}\sigma_p} \exp\left[-\frac{(p-\bar{p}_i)^2}{4\sigma_p^2}\right] \tag{3.20}$$

and

$$\psi_i(x,t) = \frac{1}{\sqrt{2\pi}\sigma_x} \exp\left[i(\bar{p}_i x - \bar{E}_i t) - \frac{(x-v_i t)^2}{4\sigma_x^2}\right] \tag{3.21}$$

where the energy $\bar{E}_i$ and the group velocity v_i are given by

$$\bar{E}_i = \sqrt{\bar{p}_i{}^2 + m_i^2}, \quad v_i = \frac{\bar{p}_i}{\bar{E}_i}, \tag{3.22}$$

and the widths σ_x and σ_p satisfy

$$\sigma_x \sigma_p = \frac{1}{2}. \tag{3.23}$$

Let us consider the two-neutrino case and a neutrino of flavor α created at $x = t = 0$ in the weak interaction prosses. The probability of finding of flavor β at x and t is given by

$$P_{\alpha\to\beta}(x,t) = \mid \sum_i U_{\beta i}^* \psi_i(x,t) U_{\alpha i} \mid^2 \tag{3.24}$$

using (3.12), we obtain for $\alpha = \nu_e, \beta = \nu_\mu$,

$$P_{\nu_e \to \nu_\mu} = \frac{1}{2\pi\sigma_x^2} \sum_{i,j} U_{\mu i}^* U_{ei} U_{\mu j} U_{ej}^* \exp[i(\bar{p}_i - \bar{p}_j)x - i(\bar{E}_i - \bar{E}_j)t] \\ \times \exp\left[-\frac{(x - v_i t)^2}{4\sigma_x^2} - \frac{(x - v_j t)^2}{4\sigma_x^2}\right] \tag{3.25}$$

$$= \frac{1}{2\pi\sigma_x} 2\sin^2\theta\cos^2\theta(1 - \cos[(\bar{p}_i - \bar{p}_j)x - i(\bar{E}_i - \bar{E}_j t)]) \\ \times \exp\left[-\frac{(x - v_i t)^2}{4\sigma_x^2} - \frac{(x - v_j t)^2}{4\sigma_x^2}\right] \tag{3.26}$$

Integlating over t, taking the relativistic limit, and using (3.23), (3.26) becomes

$$P_{\nu_e \to \nu_\mu} = 2\sin^2\theta\cos^2\theta \left\{ 1 - \cos\left(\frac{\Delta m^2}{2\bar{p}} x\right) \right. \\ \left. \times \exp\left[-\frac{x^2}{8\sigma_x^2}\left(\frac{\Delta m^2}{2\bar{p}^2}\right)^2 - (1 + \kappa)\frac{(\Delta m^2)^2}{32\sigma_p^2 \bar{p}^2}\right] \right\} \tag{3.27}$$

where

$$\kappa = \frac{\bar{p}_1^2 - \bar{p}_2^2}{\Delta m^2}. \tag{3.28}$$

When the exponential factor is close to unity, we recover neurino oscillation as given in (??)

$$P_{\nu_e \to \nu_\mu} \simeq \frac{1}{2}\sin^2 2\theta \left[1 - \cos\left(\frac{\Delta m^2}{2\bar{p}} x\right)\right] = \sin^2\theta \sin^2\left(\frac{\Delta m^2}{4E} t\right). \tag{3.29}$$

For this to happen, we need two conditions:

$$\sigma_p \gg \frac{\Delta m^2}{\bar{p}}, \tag{3.30}$$

and

$$\frac{x^2}{8\sigma_x^2}\left(\frac{\Delta m^2}{2\bar{p}^2}\right)^2 \ll 1. \tag{3.31}$$

The first condition is that the momentum should not be determined too precisely, so that it allows the transition to a different mass state.

The second condition reads,

$$x \ll \frac{\sigma_x}{\Delta m^2 / 2\bar{p}^2} \simeq \frac{\sigma_x}{|v_1 - v_2|}, \tag{3.32}$$

which means that the wave packets corresponding to the two mass states are not separated by more than the packet size.Using (3.23) the condition is also written

$$\frac{\sigma_p}{\bar{p}} \ll 2\sqrt{2}\frac{\bar{p}}{x\Delta m^2}, \tag{3.33}$$

or

$$\frac{\delta E}{\bar{E}} \ll \frac{l_0}{x}, \tag{3.34}$$

where l_0 is given by (3.16).

When either of the condition, (3.30) and (3.34), is not satisfied, the exponential function gives a damping factor in (3.27), we obtain

$$P_{\nu_e \to \nu_\mu} = 2 \sin^2 \theta \cos^2 \theta, \tag{3.35}$$

in agreement formula for time averadged oscillation (3.19). 3.2 show a oscillation probability depending of traveling length.

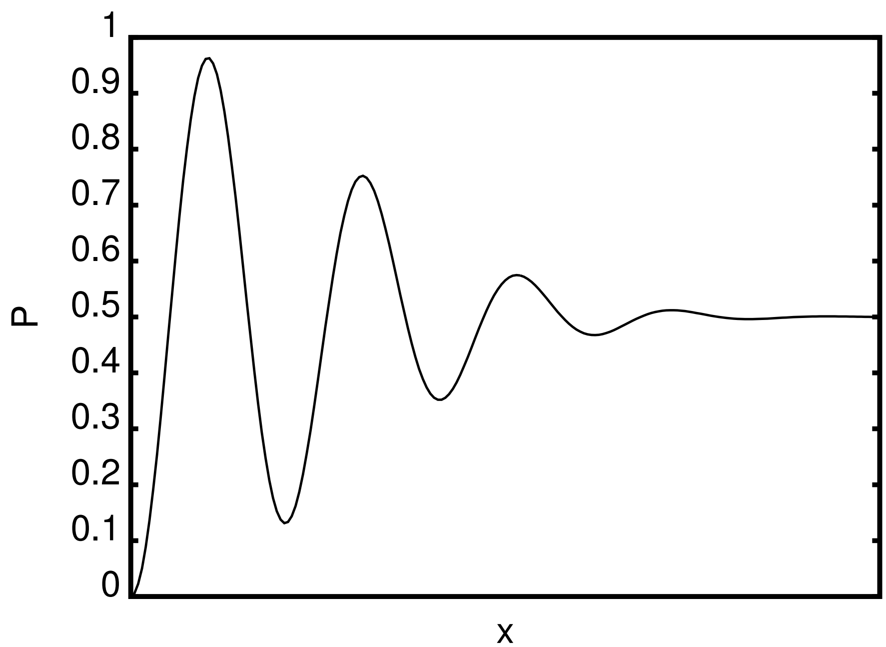

Figure 3.2: oscillation probability ν_e to ν_e after traveling long distance. This show the probability is averaged after far long distance.

In practice, the neutrinos may take a wave function with a complicated wave packet that depends on the detailed dynamics of the neutrino production process. Particularly unclear is the question of the decoherence lengthe of the neutrino waves. In a realistic case where an oscillation experiment is carried out, the size of the source is small compared with the oscillation length and the distance to the detector; neutrinos are emitted continuously from an approximate point source. With this stationary condition, one can circumvent the problem concerning the form of the wave packet and the energy eigenstates as

$$|\nu_\alpha(x,t)\rangle = \int g(E) dE e^{-iEt} \sum_{j=1}^{3} U_{\alpha j} e^{ip_j x} |\nu_j\rangle, \tag{3.36}$$

where $|\nu_j\rangle$ denote the mass eigenstates. The spectral function $g(E)$ is determined by the production process.

We assume that the neutrino just produced ($x = 0$) is in a pure weak interaction eigenstates at any time and suppose that it is ν_e. The wave function at $x = 0$ is subject to the constraint

$$\sum_{j=1}^{3} U_{ej} \langle \nu_\mu | \nu_j \rangle = \sum_{j=1}^{3} U_{ej} \langle \nu_\tau | \nu_j \rangle = 0. \tag{3.37}$$

The wave function $|\nu_\alpha(0,t)\rangle$ describes a pure ν_e state at $x = 0$ for each individual energy component.

For two neutrino oscillation, the relative phase of the neutrino wave function of ν_1 and ν_2 at distance x is

$$\delta\phi(x) = (p_1 - p_2)x = \frac{\Delta m^2}{2p} x. \tag{3.38}$$

The coherence between the two waves disappears if the phase shift $\delta\phi(x)$ varies by more than 2π across the energy width δE. Thus, the observability of neutrino oscillation is

$$\frac{\delta E}{E} < \frac{l_0}{x}, \tag{3.39}$$

which agrees with (3.34). At very large x, the wave packet separates into two packets since states with different masses have different velocities. The classical separation s is given by

$$s = \frac{\delta p}{p} x = \frac{\Delta m^2}{2p^2} x. \tag{3.40}$$

The condition $s \gg \sigma_x$ is equivalent to (3.32). There is no interference if the two wave packets are separated by more than the width of each packet.

Chapter 4

Neutrino Oscillation in Matter

4.1 Effective Potential

When neutrinos propagates through matter, ν_e and ν_μ(or ν_τ) feel different potentials because ν_e interacts with electrons via both neutral and charged current, whereas $\nu_\mu(\nu_\tau)$ interacts only via the neutral current. (Fig.4.1) These effect are either coherent or incoherent. In coherent interaction, the medium remains unchanged and it is possible to have interference of scattered and unscattered nertrino waves which enhances the effect. Coherence futher allows one to decouple the evolution equation of the neutrinos from the equations of the medium. In this approximation, the effect of the medium is described by an effective potential which depends on the density and composition of the matter.

In the presence of the effective interaction, the electron neutrino energy receives an extra contribution. Hamiltonian in the Shrödinger equation became

$$H \cong E + \frac{1}{2E}(M^2 + 2EV). \tag{4.1}$$

V is the extra potential and

$$V = V(\nu_e) - V(\nu_\mu) \tag{4.2}$$

where $V_(\nu_e)$ and $V_(\nu_\mu)$ is extra potential that interacts with ν_e and ν_μ respectively. Therefore

$$V = \begin{pmatrix} \sqrt{2}G_F n_e & 0 \\ 0 & 0 \end{pmatrix} \tag{4.3}$$

where G_F is the felumi constant and n_e is the electron number density in matter.

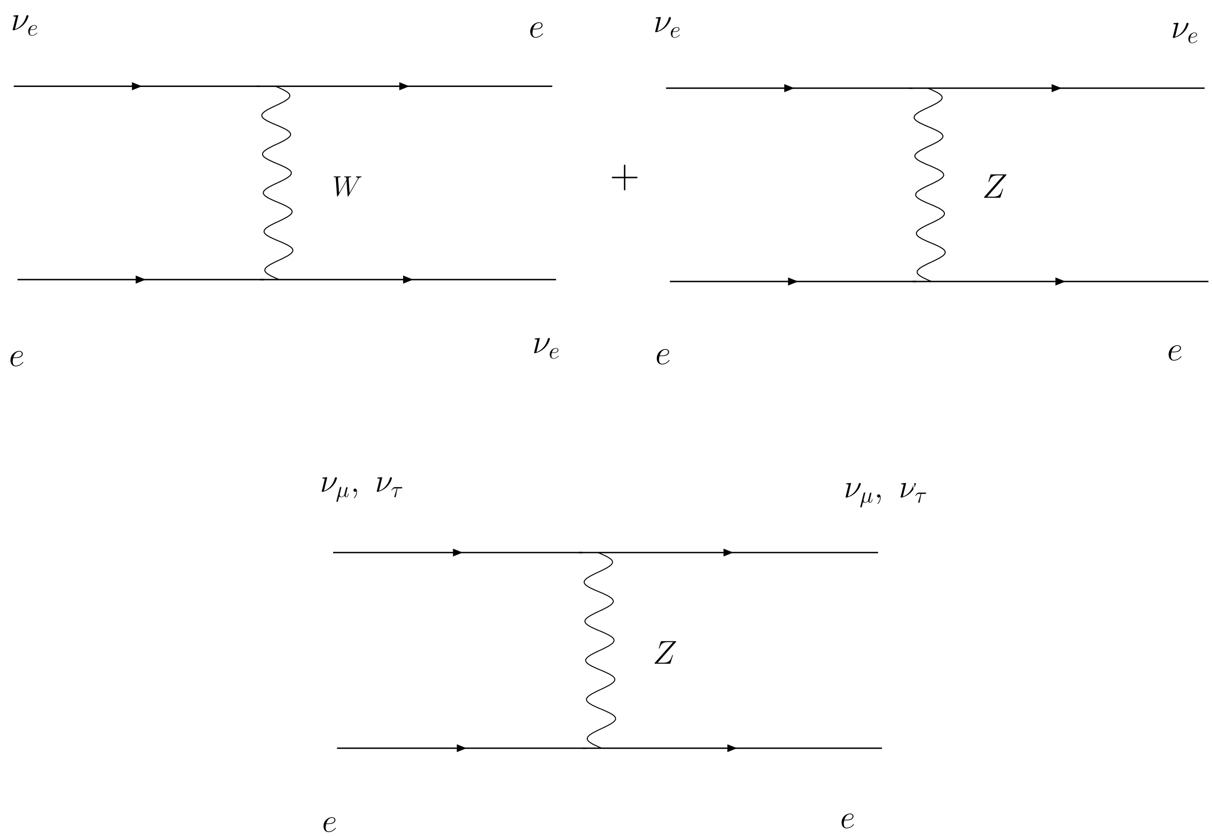

Figure 4.1: Weak interaction of neutrinos with electrons. Upper-left figure shows coherent interaction of electron neutrinos with electron via charged current interaction (W), and upper-right figure is via neutral current interaction Z. Lower figure shows coherent interaction of mu and tau neutrinos with electrons via neutral current interaction.

4.2 MSW effect

The time evolution of the neutrino wave function is given by

$$i\frac{d}{dt}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = \begin{pmatrix}-\frac{\Delta m^2}{4E}\cos 2\theta + \sqrt{2}G_F n_e & \frac{\Delta m^2}{4E}\sin 2\theta\\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta\end{pmatrix}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} \tag{4.4}$$

where n_e is independent of time. We diagonalize the hamiltonian of (4.4) by

$$\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = U_m\begin{pmatrix}\tilde{\nu}_1\\ \tilde{\nu}_2\end{pmatrix} = \begin{pmatrix}\cos\tilde{\theta} & \sin\tilde{\theta}\\ -\sin\tilde{\theta} & \cos\tilde{\theta}\end{pmatrix}\begin{pmatrix}\tilde{\nu}_1\\ \tilde{\nu}_2\end{pmatrix} \tag{4.5}$$

where $\tilde{\nu}$ is the energy eigenstate in matter and $\tilde{\theta}$ is a mixing angle in matter. The time evolution equation is

$$i\frac{d}{dt}\begin{pmatrix}\tilde{\nu}_1\\ \tilde{\nu}_2\end{pmatrix} = \Lambda + \begin{pmatrix}\tilde{m}_1^2 & 0\\ 0 & \tilde{m}_2^2\end{pmatrix}\begin{pmatrix}\tilde{\nu}_1\\ \tilde{\nu}_2\end{pmatrix} \tag{4.6}$$

where Λ is a team proprtional identity matrix and $\tilde{m}_1^2$, $\tilde{m}_2^2$ is given by

$$\tilde{m}_1^2 = A + \frac{1}{2}\sqrt{(A-\Delta m^2\cos 2\theta)^2 + (\Delta m^2)^2\sin^2 2\theta} \tag{4.7}$$

$$\tilde{m}_2^2 = A - \frac{1}{2}\sqrt{(A-\Delta m^2\cos 2\theta)^2 + (\Delta m^2)^2\sin^2 2\theta}, \tag{4.8}$$

with

$$A = \sqrt{2}EG_F n_e. \tag{4.9}$$

$\tilde{\theta}$ is given

$$\cos 2\tilde{\theta} = \frac{-A/\Delta m^2 + \cos 2\theta}{\sqrt{(A/\Delta m^2 - \cos 2\theta)^2 + \sin^2 2\theta}} \tag{4.10}$$

and

$$\sin 2\tilde{\theta} = \frac{\sin 2\theta}{\sqrt{(A/\Delta m^2 - \cos 2\theta)^2 + \sin^2 2\theta}}. \tag{4.11}$$

Hence flavor eigenstate in matter is

$$|\nu_\alpha\rangle = \sum_i (U_m)_{\alpha i}\,|\tilde{\nu}\rangle \tag{4.12}$$

as the section 3-1, time evolution of flavor eigenstate is

$$|\nu_\alpha\rangle_t = \sum_i (U_m)_{\alpha i} e^{(\Lambda + m_i^2/4E)t}\,|\tilde{\nu}_i\rangle. \tag{4.13}$$

For example, the trantision probability ν_e to ν_e or ν_e to ν_μ in matter at a distance L from the source is

$$P_{\nu_e\to\nu_e} = 1 - \sin^2\tilde{\theta}\sin^2\frac{\Delta\tilde{m}^2}{4E}L \tag{4.14}$$

$$P_{\nu_e\to\nu_\mu} = \sin^2\tilde{\theta}\sin^2\frac{\Delta\tilde{m}^2}{4E}L. \tag{4.15}$$

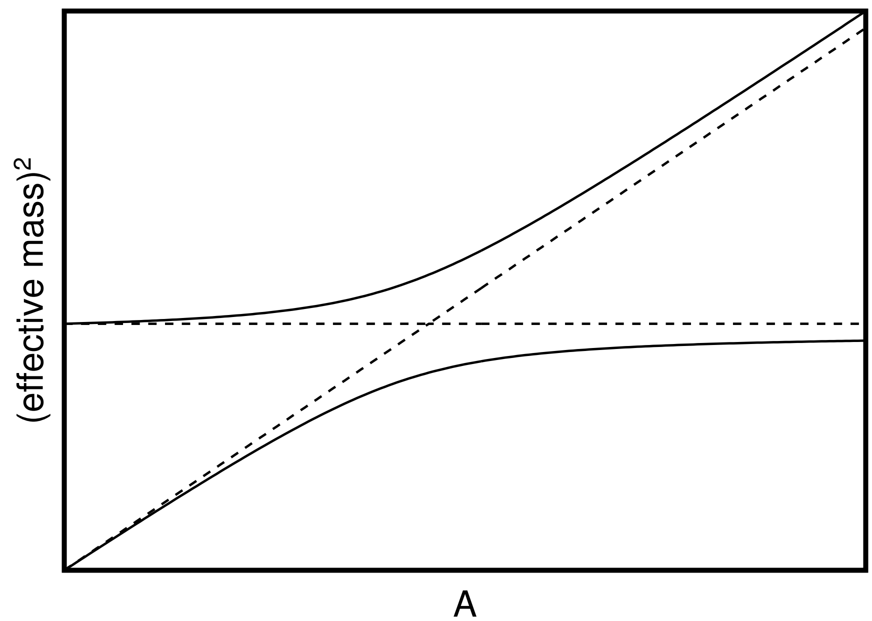

Figure 4.2: Effective neutrino mass squared in the medium with the term proportional electron density, A ,is the crossing at the 'resonance point'

We plot $\tilde{m}^2$ and $\tilde{\theta}$ as a function of n_e in 4.2.

The level crossing ('resonance') occurs at $A/\Delta m^2 = \cos 2\theta$. We define the dencity $n_{e,crit}$ and it satisfy

$$n_{e,crit} = \frac{1}{2\sqrt{2}G_F}\frac{\Delta m^2}{E}\cos 2\theta. \tag{4.16}$$

As can be seen in Fig.??, if $n_e \gg n_{e,crit}, \tilde{\theta} \simeq \theta$, and neutrino oscillate with a mixing length in matter of $l_0 = 4\pi E/\tilde{m}^2$, as in vaccume oscillation. For $n_e \ll n_{e,crit}$, $\tilde{\theta}$ approches $\pi/2$. The mixing length in matter,

$$\tilde{l} = \frac{4\pi E}{\tilde{m}_1^2 - \tilde{m}_2^2} = \frac{4\pi E}{\Delta m^2}\left[\left(\frac{A}{\Delta m^2} - \cos 2\theta\right)^2 + \sin^2 2\theta\right]^{-\frac{1}{2}} \tag{4.17}$$

is much shorter than l_0 for large n_e. At $n_e = n_{e,crit}$, two neutrinos mix maximally.

Let discuss the propagation of neutrinos. We assume that ν_e is produced at t_i ,passes through the resonance at t_r and is detected at $t_f \to \infty$ in vacuum. At $t = t_i$,

$$\psi_{\nu_e}(t_i) = \cos\tilde{\theta}\,|\tilde{\nu_1}, t_i\rangle + \sin\,|\tilde{\nu_2}, t_i\rangle, \tag{4.18}$$

accoding to (4.5). At $t = t_r - \epsilon$, $\psi(t)$ can be written by using the state at $t = t_r$,

$$\psi(t_r - \epsilon) = \cos\tilde{\theta}e^{i\int_{t_i}^{t_r}\varepsilon_1 dt}\,|\tilde{\nu_1}, t_i\rangle + \sin\tilde{\theta}e^{i\int_{t_i}^{t_r}\varepsilon_2 dt}\,|\tilde{\nu_2}, t_r\rangle, \tag{4.19}$$

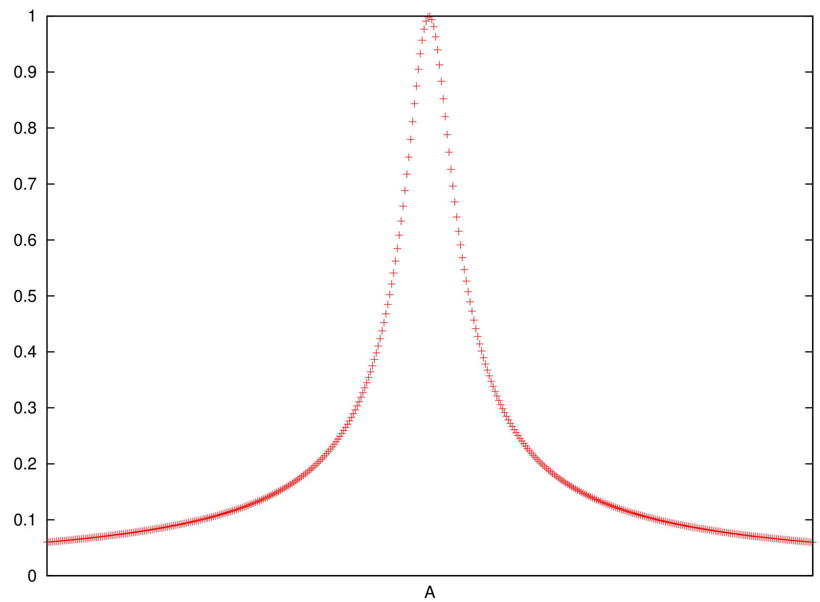

Figure 4.3: $\sin^2 2\tilde{\theta}$ in A. It became maximall at the resonance point and oscillation probability also is maximall at the point.

where $\varepsilon_{1,2} = \tilde{m}^2_{1,2}/2E$. A flip to the other state may take place at the resonance point and the state is written as

$$|\tilde{\nu_1}, t_r\rangle \longrightarrow \alpha\,|\tilde{\nu_1}, t_r\rangle + \beta\,|\tilde{\nu_2}, t_r\rangle \tag{4.20}$$

$$|\tilde{\nu_2}, t_r\rangle \longrightarrow -\beta^*\,|\tilde{\nu_1}, t_r\rangle + \alpha^*\,|\tilde{\nu_2}, t_r\rangle \tag{4.21}$$

with $|\alpha|^2 + |\beta|^2 = 1$. After the resonance point,

$$\psi(t_r + \epsilon) = \cos\tilde{\theta} e^{i\int_{t_i}^{t_r} \varepsilon_1 dt}(\alpha\,|\tilde{\nu_1}, t_r\rangle + \beta\,|\tilde{\nu_2}, t_r\rangle) \tag{4.22}$$

$$+ \sin\tilde{\theta} e^{i\int_{t_i}^{t_r} \varepsilon_2 dt}(-\beta^*\,|\tilde{\nu_1}, t_r\rangle + \alpha^*\,|\tilde{\nu_2}, t_r\rangle) \tag{4.23}$$

$$\equiv A(t_r)\,|\tilde{\nu_1}, t\rangle + B(t_r)\,|\tilde{\nu_2}, t\rangle. \tag{4.24}$$

For $t > t_r$, it propagates as

$$\psi(t) = A(t_r) e^{i\int_{t_r}^{t} \varepsilon_1 dt} + B(t_r) e^{i\int_{t_r}^{t} \varepsilon_2 dt}. \tag{4.25}$$

At $t_f = \infty$,

$$\psi(\infty) = A(t_r) e^{i\int_{t_r}^{\infty} \varepsilon_1 dt}(\cos\theta\,|\nu_e\rangle - \sin\theta\,|\nu_\mu\rangle) \tag{4.26}$$

$$+ B(t_r) e^{i\int_{t_r}^{\infty} \varepsilon_2 dt}(\sin\theta\,|\nu_e\rangle) + \cos\theta\,|\nu_\mu\rangle, \tag{4.27}$$

where $|\tilde{\nu}_i, \infty\rangle = |\nu_i\rangle$ and $|\nu_i\rangle = U^\dagger |\nu^\alpha\rangle$ are used. Then,

$$\langle \nu_e | \psi(\infty) \rangle = A(t_r) e^{i \int_{t_r}^{\infty} \varepsilon_1 dt} + B(t_r) e^{i \int_{t_r}^{\infty} \varepsilon_2 dt} \tag{4.28}$$

and

$$P_{\nu_e \to \nu_e} = |A|^2 \cos^2\theta + |B|^2 \sin^2\theta \qquad +2|AB| \cos\theta \sin\theta \cos\left[\int_{t_r}^{\infty} (\varepsilon_1 - \varepsilon_2) dt + \Omega\right] \tag{4.29}$$

with $\Omega = \arg A^* B$. The last term vanishes if we take an average over the detector position (or the beam energy spread). Inserting expression for $A(t_r)$ and $B(t_r)$, we obtain

$$\begin{aligned} < P_{\nu_e \to \nu_e} >_\infty &= |A|^2 \cos^2\theta + |B|^2 \sin^2\theta & (4.30)\\ &= \frac{1}{2} + \frac{1}{2}(|\alpha|^2 - |\beta|^2) \cos 2\tilde{\theta} \cos 2\theta & (4.31)\\ &- |\alpha\beta| \sin 2\tilde{\theta} \cos 2\theta \cos\left[\int_0^{t_r} (\varepsilon_1 - \varepsilon_2) dt + \omega\right], & (4.32) \end{aligned}$$

where $\omega = \arg \alpha^* \beta$. The last term also vanishes if we teke an average over the position of neutrino production. Identifying

$$|\beta|^2 = P_f, \tag{4.33}$$

we obtain

$$< P_{\nu_e \to \nu_e} >_{i,\infty} = \sin^2\theta + P_f \cos 2\theta. \tag{4.34}$$

If ν_e which is produced in the region $n_e > n_{e,crit}$ and propagates into the region $n_e, n_{e,crit}$, the state follows the upper branch given in 4.2. Then if the state not satisfy 'adiabatic condition' it can undergoes a trantision to the lower branch while passing through the resonance point with a probability given by P_f. Let discuss the adiabatic condition in the next chapter.

4.3 Adiabatic condition

First we calculated the transition probability, P_f, at the crossing point. Let us write (4.4) in the form

$$H\psi = \begin{pmatrix} \epsilon_1 & \epsilon_{12} \\ \epsilon_{12} & \epsilon_2 \end{pmatrix} \psi. \tag{4.35}$$

where

$$\begin{aligned} \epsilon_1 &= -(\Delta m^2/4E) \cos 2\theta + a(t), \\ \epsilon_2 &= (\Delta m^2/4E) \cos 2\theta, \\ \epsilon_{12} &= (\Delta m^2/4E) \sin 2\theta. \end{aligned} \tag{4.36}$$

Here, $a(t) = \sqrt{2} G_F n_e(t)$ is taken as a function of time. Take an orthonormal basis (c_1, c_2), the Schrödinger equation $(H - i\partial/\partial t)\psi = 0$ is written

$$\left(H - i\frac{d}{dt}\right)[c_1(t)e^{-i\int \epsilon_1 dt} + c_2(t)e^{-i\int \epsilon_2 dt}] = 0. \tag{4.37}$$

Using (4.35), (4.37)becomes

$$\begin{aligned} i\frac{dc_1}{dt} &= \epsilon_{12}e^{-i\int(\epsilon_2-\epsilon_1)dt}c_2, \\ i\frac{dc_2}{dt} &= \epsilon_{12}e^{-i\int(\epsilon_1-\epsilon_2)dt}c_1. \end{aligned} \tag{4.38}$$

The boundary condition is that ψ is in the eigenstate of $|\nu_e\rangle = |c_1\rangle$, i.e., on the upper branch in Fig.8.6 at $t \to \infty(n_e \to_i nfty)$,

$$\begin{aligned} |c_1(-\infty)| &= 1 \\ c_2(-\infty) &= 0. \end{aligned} \tag{4.39}$$

The problem is to find the probability that ψ jumps from the upper to the lower branch in the resonance region, i.e., that ψ is in the state $|\nu_e\rangle = |c_1\rangle$ at $t \to +\infty$,

$$P = |c_1(\infty)|^2 = 1 - |c_2(\infty)|^2. \tag{4.40}$$

We now eliminate c_1 from (4.38),

$$\frac{d^2c_2}{dt^2} + i(\epsilon_1 - \epsilon_2)\frac{dc_2}{dt} + \epsilon_{12}^2 c_2 = 0. \tag{4.41}$$

We assume that $n_e(t)$ varies linealy in t in the resonance region, which is taken as $t = 0$:

$$n_e(t) = n_e^0 + \dot{n_e^0}t, \tag{4.42}$$

where n_e^0 satisfies the resonance condition, $\sqrt{2}G_F n_e^0 = (\Delta m^2/2E)\cos 2\theta$. Writing

$$\epsilon_1 - \epsilon_2 = \sqrt{2}G_F\dot{n_e^0}t \equiv \alpha t, \tag{4.43}$$

and putting

$$c_2(t) = e^{-\frac{i}{2}\int(\epsilon_1-\epsilon_2)dt}U(t), \tag{4.44}$$

(4.41) reads

$$\frac{d^2U}{dt^2} + \left[\epsilon_{12}^2 - \frac{i}{2}\alpha + \frac{1}{4}(\alpha t)^2\right]U = 0. \tag{4.45}$$

Upon changing variables,

$$z = \sqrt{\alpha}e^{-i\pi/4}t, \tag{4.46}$$

and

$$n = i\epsilon_{12}^2/\alpha, \tag{4.47}$$

(4.45) becomes the Weber equation,

$$\frac{d^2U}{dz^2} = \left(n + \frac{1}{2} - \frac{z^2}{4}\right) U = 0. \tag{4.48}$$

The solution is called the Weber function, $D_{-n-1}(\pm iz)$. Because $D_{-n-1}(iz)$ behaves asymptotically, $D_{-n-1}(iz) \to 0$ as $z \to \infty e^{-\frac{3}{4}\pi i}$ $z \to \infty e^{-\frac{\pi}{4}i}$ $(t \to \infty)$, and not satisfies the boundary condition of c_2,

$$U = A_+ D_{-n-1}(-iz) \tag{4.49}$$

satisfies the required boundary condition, $c_2 \sim U \to 0$, as $t \to -\infty$. Here, A_+ is the normalisation factor that is determined by $|c_1(-\infty)| = 1$. Defining $R = -\sqrt{\alpha}t$ and using the second equation of(4.38) and the asymptotic behaviour of $D_{-n-1}(iRe^{-\pi i/4})$ as $R \to \infty (t \to -\infty)$,

$$D_{-n-1}(iRe^{-\frac{\pi}{4}i}) \sim e^{\frac{\pi}{4}(n+1)i} e^{-\frac{R^2}{4}i} R^{-n-1}, \tag{4.50}$$

we have

$$c_1(\infty) = \frac{\sqrt{\alpha}}{\epsilon_{12}} A_+ e^{-\frac{\pi}{4}\nu} e^{-\frac{3}{4}\pi i - i\nu \ln(-\sqrt{\alpha}t)}. \tag{4.51}$$

The normalisation condition is given by

$$|A_+| = \sqrt{\nu} e \, -\frac{\pi}{4}\nu, \tag{4.52}$$

where

$$\nu = -in = \epsilon_{12}^2/\alpha. \tag{4.53}$$

For $t \to +\infty$, we pick up the leading term of the asymptotic expansion of $D_{-n-1}(iRe^{3\pi i/4})$as $R \to \infty$,

$$D_{-n-1}(iRe^{3\pi i/4}) \sim \frac{\sqrt{2\pi}}{\Gamma(n+1)} e^{\frac{\pi}{4}in} e^{\frac{R^2}{4}i} R^n. \tag{4.54}$$

Then,

$$|c_2(\infty)|^2 = \nu e^{-\frac{\pi}{2}\nu} \frac{2\pi}{\Gamma(i\nu+1)\Gamma(-i\nu+2)} e^{-\frac{\pi}{2}\nu}. \tag{4.55}$$

Using the relation for a real number ν

$$\Gamma(i\nu) = \left[\frac{\pi}{\sinh(2\pi)}\right]^{\frac{1}{2}}, \tag{4.56}$$

then,

$$\begin{aligned} |c_2(\infty)|^2 &= 2\sinh(\pi\nu) e^{-\pi\nu} \\ &= 1 - e^{-2\pi\nu}, \end{aligned} \tag{4.57}$$

where

$$\nu = \frac{1}{2}\frac{\Delta m^2}{2E}\frac{\sin^2 2\theta}{\cos 2\theta}\left(\frac{d\ln n_e}{dr}\right)^{-1} \equiv \frac{1}{4}\gamma \tag{4.58}$$

from (4.36),(4.43),(4.47) and the resonance condition. Therefore, we obtain

$$P_f = \exp\left(-\frac{\pi}{2}\gamma\right), \tag{4.59}$$

We call this Landau-Zener formula. Note that this formula can applies when the variation in density is linear in r and is acclate unless the mixing angle is close to $4/\pi$, which corresponds to zero matter density.

...

Note here that the neutrinos emergent from the Sun are in the eigenstate of mass, irrespective of whether or not conversion takes place, ucless the mass difference is very small.This is clear from the fact that $\nu_e = \tilde{\nu_2}$ for a high density region; the state follows the branch all the way adiabatically, or does so before and after a flip in the resonance region, and the two branches continue smoothly to ν_2 or ν_1 at the surface of the Sun. These neutrinos emargent from the Sun hence do not undergo further oscillation in vaccum oscillation.

The exception is when the neutrino mass difference is so small that the resonance condition is satisfied only close to the sutface of the Sun; when the oscillation length becomes longer than the distance between the rasonance position and the surface, neutrinos do not fall into mass eigenstates. In this case, oscillation in vaccume may take place, as in the 'jus-so' scemario.

In summary, complete conversion of ν_e to ν_2 takes place in the Sun when two conditions are satisfied: (1) in the centre of the Sun, $n_e > n_{e,crit}$, which leads to $\Delta m^2 \leq const$ (resonance condition); and (2) $\Delta m^2 \sin^2 2\theta \geq const$, as derived from (??) (when θ is not too close to 45) (adiabatic condition). In addition, (??) shows that $P_{\nu_e \to \nu_e} = \frac{1}{2}$ at $\sin^2 2\theta = 1(\gamma = \infty)$ independent of Δm^2 and energy. This means that the $P_{\nu_e \to \nu_e} = const(\leq \frac{1}{2})$ forms approximately a rectangular triangle (which we reffer to as the MSW triangle) in the $(\Delta m^2, \sin^2 2\theta)$ plane.

Chapter 5

Present Status of Experiments and Observation

The nuclear processes in the Sun make only ν_e, not ν_μ or ν_τ. Neutrinos emitted in the Sun are electron neutrinos produced in the thermonuclear reactions which generate the solar energy. In Standard Ssolar Model (SSM), which the most updated version of the Sun model, these reactions occer via two main chains, the pp-chain 5 and the CNO cycle. There are five reactions which produce ν_e in the pp chain and three in the CNO cycle.In particulaly, electron neutrino criated by β decay of ^8B in the pp-chain is used expriment because it have large energy.

We introduce a recently experiments. The suggestion that neutrino oscillation may be a realistic possibility was first made by Pontecorvo in relation to the early solar neutrino experiment of Davis.Jr. and collaborators in Homestake by using (^{37}Cl and the reaction

$$\nu_e + \ ^{37}\mathrm{Cl} \ = \ e^- + \ ^{37}\mathrm{Al} \ . \tag{5.1}$$

The energy threshold for this reaction is 0.814 MeV, so the relevant fluxes are the ^{7}Be and ^{8}B. The average event rate measured during the more than 20 years of operation is

$$R_{Cl} = 2.56 \pm 0.16 \pm 0.16 \ \ \mathrm{SNU} \ , \tag{5.2}$$

(1SNU $= 10^{-36}$ captures/atom/sec) which corresponds to approximately one third of the SSM prediction. It indicated that the observed solar electron neutrino flux was smaller than theoretical calculations from the Standard Solar Model (SSM). This is the first evidenceof solar neutrino ploblem.

In January 1990 and May 1991, two new radiochemical experiments using a $^71textGa$ target started takeing data, SAGE in Baksan, Russia and GALLEX in Gran Sasso, Italy. In these experiments the solar neutrinos are captired via

$$\nu_e +^{71}\mathrm{Ga} \rightarrow e^- +^{71}\mathrm{Ge} \tag{5.3}$$

reaction	separation rate	ν_e energy(MeV)
p+p $\rightarrow$ ^{2}H + e^+ + ν_e	100	$\leq$ 0.420
or		
p+e^-+p $\rightarrow$ ^{2}H+ν_e	0.4	1.442
^{2}H+p $\rightarrow$ ^3He + γ	100	
^3He + ^{3}He $\rightarrow$ α+2p	85	
or		
^{3}He+ ^{4}He $\rightarrow^7$ Be + γ	15	
^{7}Be + e^- $\rightarrow^7$ Li + ν_e	15	(90%)0.861
		(10%)0.383
^{7}Li + $p \rightarrow 2\alpha$	15	
or		
^{7}Be + $p \rightarrow^8$ B + γ	0.02	
^{8}B $\rightarrow^8$ B* + e^+ + ν_e	0.02	<15
or		
^{3}He + $p \rightarrow^4$ He + e^+ + ν_e	0.00002	<18.77

Table 5.1: pp-chain reaction in the Sun

According to the SSM, approximately 54% of the events are due to *pp* neutrinos, while 26% and 11% arise from ^{7}Be and ^{8}B neutrinos respectively. The event rates measured by SAGE and GALLEX are

$$R_{SAGE} = 70.8^{+5.3}_{-5.2}\,{}^{+3.7}_{-3.2} \text{ SNU} \tag{5.4}$$

$$R_{GALLEX} = 77.5 \pm 6.2^{+4.3}_{-4.7} \text{ SNU} \tag{5.5}$$

while the prediction of the SSM is 130 SNU.

Kamiokande and its successor SupreKamiokande (SK) in Japan are water Cerenlov detectors that are able to detect in real time the electrons scatterd from the water by elastic interaction of the solar neutrinos,

$$\nu_\alpha + e^- \rightarrow \nu_\alpha + e^- \ , \tag{5.6}$$

The ecattered electrons produce Cerenkov light which is detected by photomultipliners. Kamiokande, with 2140 tons of water, started taking data in January 1987 and was terminated in February 1995. SK, with 45000 tons of water (of which 22500 are usable in solar neutrino measurments) started in May 1996 and it has analyzed so far the events corresponding to 1258 days. The detection threshold in Kamiokande was 7.5 MeV while SK started at a 6.5 MeV threshold and is currently running at 5 MeV. This means that these experiments are able to measur only the ^8textB neutrinos (and the vely small hep neutrino flux). Their results are presented in terms of measured ^{8}B flux.

The final result of Kamiokande and the latest result of SK are

$$\phi_{Kam} = (2.80 \pm 0.19 \pm 0.33) \times 10^6 cm^{-2}s^{-1} \ , \phi_{SK} \quad = (2.35 \pm 0.02 \pm 0.08) \times 10^6 cm^{-2}s^{-1} \ , \tag{5.7}$$

corresponding to about $40 - 50\%$ of the SSM prediction.

There are three features unique to the water Cerenkov detectors. First, they are real time expetiments. Each event is indevidually recorded. Second, for each event the scattered electron keeps the neutrion direction within an angular interval which depends on the neutrino energy as $\sqrt{2m_e/E_\nu}$. Thus, it is possible, for example, to correlate the neutrino detection with the position of the Sun. Third, the amount of Cerenkov light produced by the scattered electron allows a measurment of its energy. In summary, the experiment provides information on the time, direction and energy for each event.

The Sudbury Neutrino Observatory (SNO) was first proposed in 1987 and it started taking data in November 1999. The detector, a great sphere surrounded by photomultipliners, contains approximately 1000 tons of heavy water, D_2O and is located at the Creighton mine, near Sudbury in Canada. SNO was designed to give a model independent test of the possible explanations of the obserbed deficit in the solar neutrino flux by having sensitivity to all flavors of active netrinos and not just to ν_e

This sensitivity is achieved because energetic neutrinos can interact in the D_2O of SNO via three different reactions. Electron neutrinos may interact via the Charged Current (CC) reaction

$$\nu_e + d \rightarrow p + p + e^- \ , \tag{5.8}$$

and can be detected above an energy threshold of a few MeV. All active neutrinos ($\nu_\alpha = \nu_e, \nu_\mu, \nu_\tau$) interact via the Neutral Current (NC) reaction

$$\nu_\alpha + d \rightarrow n + p + \nu_\alpha \ , \tag{5.9}$$

with an energy threshold of 2.225 MeV. The non-sterile neutrinos can also interact via Elastic Scattering (ES), $\nu_\alpha + e^- \rightarrow \nu_\alpha + e^-$, but with smaller cross section. In June 2001, SNO published their first result. Its spectrum is the result of the combination of three possible sugnals. Assuming an undistorted energy spectrum they extract the indevidual rates:

$$\begin{aligned}
\Phi^{CC}_{SNO} &= (1.76^{+0.06}_{-0.05} \pm 0.09) \times 10^6 cm^{-2}s^{-1} \\
\Phi^{ES}_{SNO} &= (2.39 \pm 0.24 \pm 0.12) \times 10^6 cm^{-2}s^{-1} \\
\Phi^{NC}_{SNO} &= (5.09^{+0.44+0.46}_{-0.43-0.43}) \times 10^6 cm^{-2}s^{-1} \ .
\end{aligned} \tag{5.10}$$

5.1 (a) is the each result of Homestake, SAGE,GALLEX,and SNO and (b) is the combined fit. there are several solutions. Small mixing solution (SMA), Large mixing solution (LMA), or Low solution but recently experiments , for instance in Kamland which using liquid scintilaotr 5.2, show an evidence that the solution is LMA and there are also three solutions in the LMA ragion.

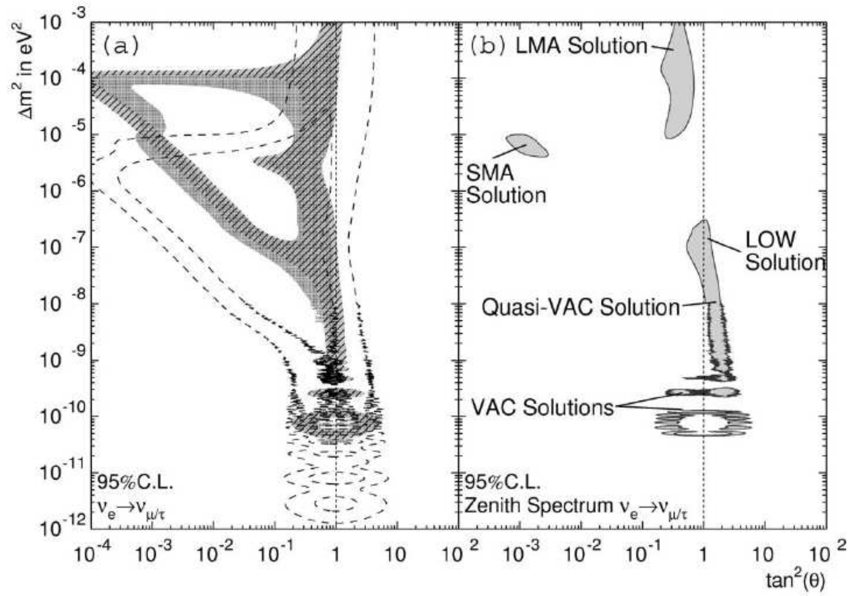

Figure 5.1: (a)Allowed areas from three experiments measuring the solar ν_e flux. The thaded area uses only Homestake data, while the hatched area uses only the SNO charged-current rate. Overlaid (inside dashed lines) is the region allowed by Gallex/GNO and SAGE. (b)Allowed regions from a conbined fit to these charged-current rates. All conturs in this and other figures are 95% C.L.

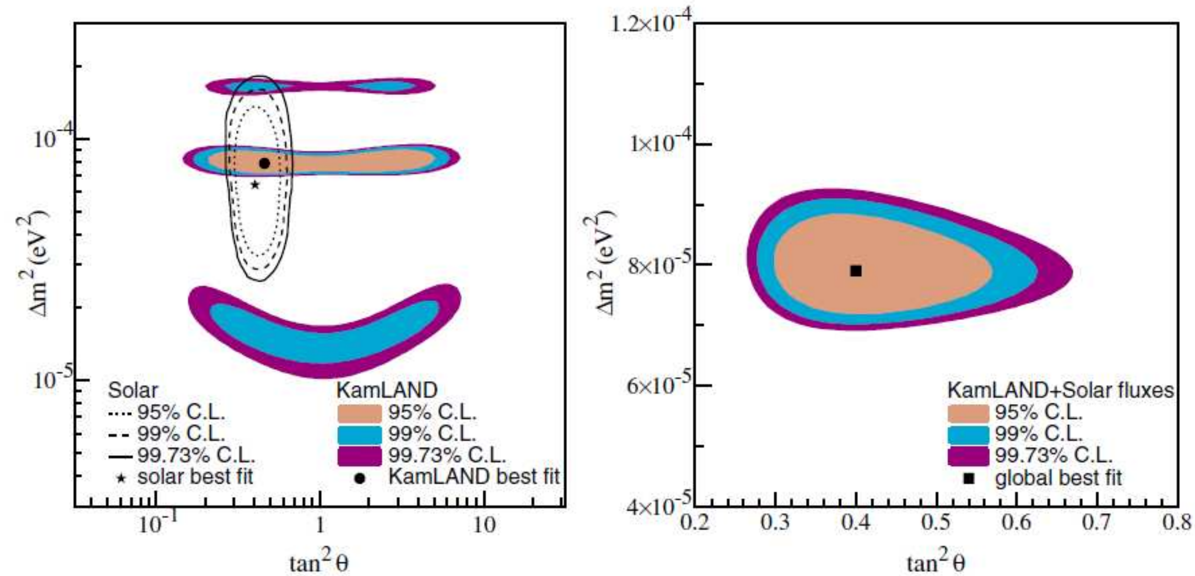

Figure 5.2: (a)Neutrno oscillation parameter allowed region from KamLAND antineutrino data (shaded ragions) and solar-neutrino experiments (lines) (b)Result of a combined two neutrino oscillation analysis of KamLAND and obserbed solar-neutrino fluxes.

Chapter 6

Summary

I showed that neutrino oscillation is the evidence of having neutrino mass and solar neutrino experiments show an evidence of the phenomenon. Also I showed the appropriation of the MSW effect to the solar neutrino problem. Oscillation parameters are in LMA region by a recently experiments. Today, only data whice is can not explanated by the Standard Model is having neutrino mass. We can expect the new physics.

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